Title: Isometries: Walkin' & Talkin', Movin' & Groovin'

Brief Overview:

Students will use graphing calculators and CBL's with the motion detector probes to explore isometric transformations. First, they will create an initial curve and illustrate the four isometries by collecting data while walking. Next, using a right scalene triangle, they will investigate isometries in coordinate geometry.

Links to Standards:

- **Mathematics as Problem Solving**
  The students will demonstrate their ability to transform curves and triangles using reflection, translation, rotation, and glide reflection.

- **Mathematics as Communication**
  The students will discuss and conjecture the procedures necessary to obtain the various transformations. Furthermore, they will describe, in writing, the process that gave an acceptable outcome.

- **Mathematics as Reasoning**
  The students will make and test conjectures involving motion with transformations and coordinate geometry.

- **Mathematical Connections**
  The students will recognize the relationship between the physical motion and the graphical models. Algebra and geometry will be inherently be linked in this activity.

- **Geometry from an Algebraic Perspective**
  The students will use the coordinate plane to represent various transformations. Further work may be done using ordered pairs, vectors, and distances.

Links to Maryland High School Mathematics Core Learning Goals:

- **2.1.1** The student will describe the characteristics of geometric figures and will construct or draw geometric figures using technology and tools.

- **2.1.3** The student will use transformations to move figures, create designs, and demonstrate geometric properties.

Grade/Level:

Grades 9 - 12, Geometry

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Sketching transformations / isometries
- Identifying ordered pairs in a coordinate plane
- Using appropriate vocabulary for triangles and transformations

Objectives:

Students will:

- work cooperatively in groups.
- use the technical equipment necessary to perform the activity.
- create and transform a curve and a scalene triangle.
- express their understanding of the four isometries in verbal and written form.

Materials/Resources/Printed Materials:

Per group:

- TI-82 or TI-83 graphing calculator (2)
- CBL and link cable (2)
- Motion detector (2)
- Dowel rod, about 1/4” thick and 12” long (1)
- Meter/yard stick (1)
- Tape
- Student Activity Worksheets
- MOTION and PLOTS programs on the calculator from Exploring Physics and Math with the CBL System, Brueningsen, et al (1994)
- DISTFORM program on the calculator from Real-World Math with the CBL System, Brueningsen, et.al. (1994)

Development/Procedures:

Activity #1:

Each group of 3 or 4 students will need only one calculator, CBL, motion detector, link cable (press ends in firmly) and a set of worksheets. A TI-ViewScreen either for the overhead or TV could be used for a demonstration. Load the MOTION and PLOTS programs on each calculator. (If CBL does not recognize motion detector, try running MOTION-RT program from same source.) Students may need guidance through set up and creating the initial image as directed in the activity sheets. The amount of time for the data collection may be adjusted. A simple ‘S’ or ‘U’ curve may be the best choice for the initial image.

Activity #2:

For this activity each group will need 2 sets of equipment and sufficient table/floor space. Refer to Activity 1 from Real-World Math with the CBL System, Brueningsen, et.al. (1994) for a picture of the set up and information on step by step procedures. Load the program DISTFORM into each calculator. Students may need guidance through set up and movement of the pattern sheet.

Performance Assessment:

Teacher will circulate among the groups to ensure that students are on task. Groups could be evaluated based on performance and completion of activity sheets. Special attention should be given to students’ descriptions and predictions.

Extension/Follow Up:

- Each activity could be extended to include dilations and symmetry. In Activity #1 the data collection time could be varied or use one of the other graphing options in the program MOTION. In Activity #2 the pattern sheet could be redesigned by the teacher or the student.
- Follow up activities include: symmetry, dilation, paper folding, frieze patterns, kaleidoscopes, logos, Escher art, etc.

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Assessment
(Activity #1)

1. Describe the WALKER'S movements that created the the image in figure 1.

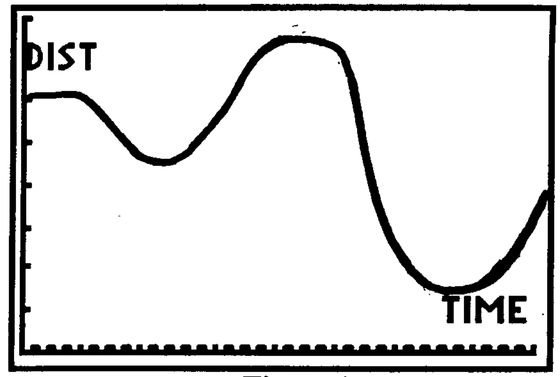
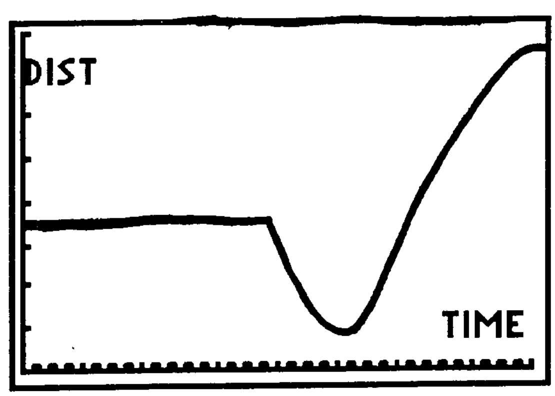


Figure 1

2. If the previous image was created using a walking time of 20 seconds and the reflection line is x = 15, draw a sketch of the reflection on figure 1. Describe the WALKER'S movements to create this reflection image.

Rubric for Activity #1 Assessment

1.	4	Accurate in describing all distances to detector
	3	Generally correct in describing distances to detector
	2	Response indicates that curve is determined by distance to detector
	1	An attempt to respond
	0	No response
2.	4	Accurate reflection graph with horizontal line to beginning of WALKER'S movement and description gives distances to detector
	3	Graph indicates correct reflection but reflection line incorrect or no horizontal line at beginning. Description generally accurate.
	2	Some errors in reflection graph. Description of movement is vague.
	1	Graph and description attempted.
	0	No response.



Activity #1

Name____________________ Name____________________
Name____________________ Name____________________

Set up equipment.

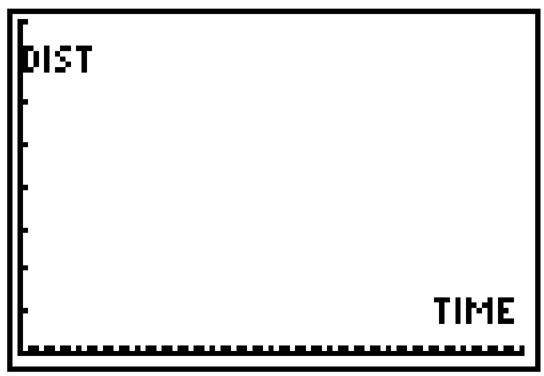
- Connect CBL to the calculator using a unit-to-unit link and turn on each unit.
- Connect motion detector to the SONIC port on the left side of the CBL unit.
- Place the motion detector on a table perpendicular to the line of motion of a walker.
- Clear a 20-24 ft pathway in front of the detector.
- Locate and use ENTER to pull up the program MOTION from the program menu on the calculator.

Create initial image.

- One student will stand about 1.5 feet from the motion detector. This student will be referred to as the WALKER.
- When another student activates the graph in the MOTION program, the WALKER will walk away from the motion detector (up to 20 ft) and then back toward the motion detector. Select the walking time to be 10 seconds.
- Use plot option #1 distance-time to see your curve on the calculator screen.
- Several attempts may be made to get a smooth curve which will be called the initial image.
- When you are happy with your graph, SKETCH YOUR INITIAL IMAGE ON GRAPHS 1 - 5.

Graph #1

Initial Image



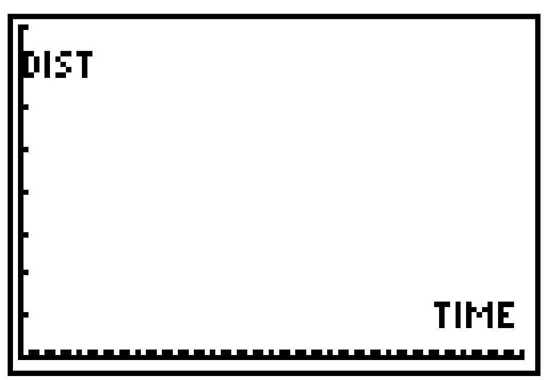
Determine and walk to create the reflection image.

- Discuss how to create a reflection image by walking.
- Choose a horizontal line to be used as the line of reflection. Write the equation of the line and draw the line on the graph below.

Graph #2

Reflection Image

line of reflection: y = _____



Activity #1

- Continue using the MOTION program to create the reflection image. Several attempts may be necessary.
- Sketch your reflection image on Graph #2 using a different color .
- Describe the WALKER's movements to get the reflection image.

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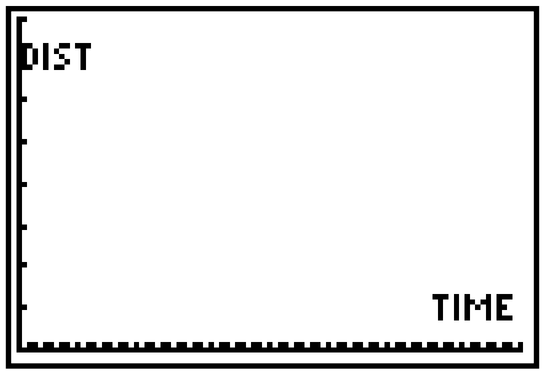
Determine and walk to create the translation image.

- Discuss how to create a translation image by walking.

Graph #3

Translation Image



- Continue using the MOTION program to create the translation image. Several attempts may be necessary.
- Sketch your translation image on Graph #3 using a different color.
- Describe the WALKER's movements to get the translation image.

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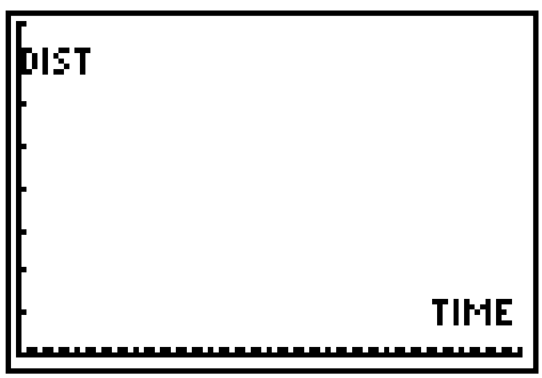
Determine and walk to create the rotation image.

- Discuss how to create a 30 degree rotation image about the origin by walking.

Graph #4

Rotation Image



Activity #1

- Continue using the MOTION program to create the rotation image about the origin. Several attempts may be necessary.
- Sketch your rotation image on Graph #4 using a different color.
- Describe the WALKER's movements to get the rotation image.

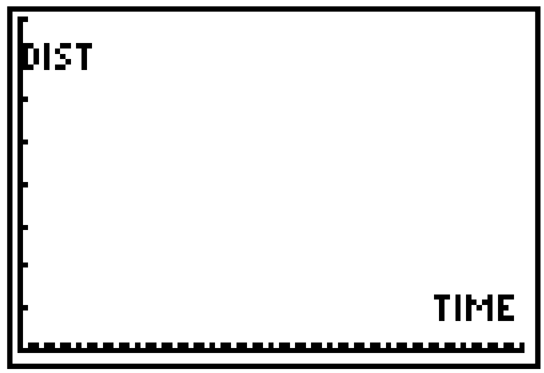
Determine and walk to create the glide reflection image.

- Discuss how to create a glide reflection image by walking.
- Choose a horizontal line to be used as the line of reflection. Write the equation of the line and draw the line on the graph below.

Graph #5

Glide Reflection Image

line of reflection: y = ______



- Continue using the MOTION program to create the glide reflection image. Several attempts may be necessary.
- Sketch your reflection image on Graph #5 using a different color.
- Describe the WALKER's movements to get the glide reflection image.

Activity #2

Name____________________ Name____________________
Name____________________ Name____________________

Set up the equipment.

- Connect CBL to the calculator using a unit-to-unit link and turn on each unit.
- Connect motion detector to the SONIC port on the left side of the CBL unit.
- Connect a second set of equipment.
- Tape the pattern sheet to a table/floor noting space requirements.
- Tape the motion detectors as indicated on the pattern sheet.
- Locate and use ENTER to pull up the program DISTFORM from the program menu on the calculator.

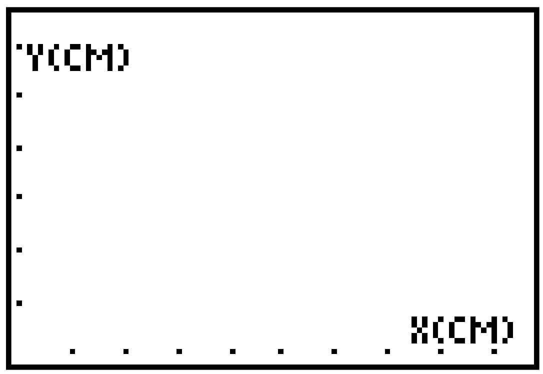
Create the pattern image.

- Activate and follow the DISTFORM program . Use #1 collect data. View instructions to verify the set up.
- Note from pattern sheet which calculator is X and which is Y.
- With all students standing out of the path of the motion detectors, one student will trace the triangle using the dowel rod in a vertical position.
- Several attempts may be made to get a triangle which will be called the pattern image.
- When you are happy with your graph, sketch your pattern image on **graphs #1 - 5**.
- Use TRACE to identify the coordinates of each vertex to the nearest tenth.

Graph #1

Pattern Image

Coordinates:
S ______
A______
M______



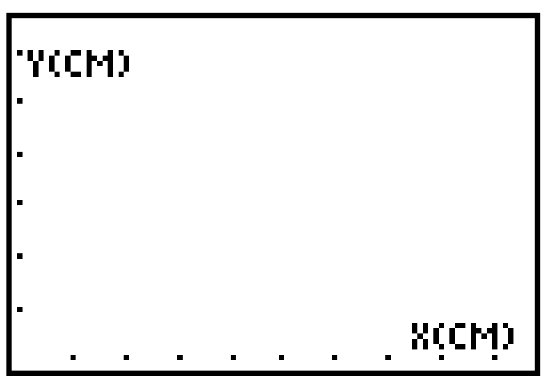
Create the reflection image.

- Discuss how to create a reflection image by moving, flipping and/or sliding the pattern.
- Choose a horizontal line to be used as the line of reflection. Write the equation of the line and draw the line on the graph below.

Graph #2

Reflection Image

line of reflection: y = _____



Activity #2

- Continue using the DISTFORM program to create the reflection image. Several attempts may be necessary.
- Sketch your reflection image on Graph #2 using a different color .
- Label the corresponding vertices S’, A’, and M’.
- Use TRACE to identify the coordinates of the new vertices to the nearest tenth.

Coordinates: S’_______   A’________   M’________

- Describe the movement of the pattern to get the reflection image and compare the corresponding coordinates.

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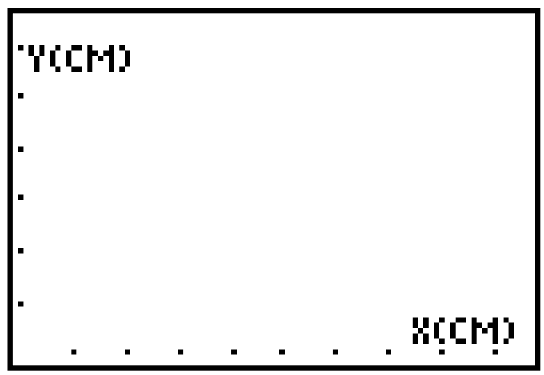
Create the translation image.

- Discuss how to create a translation image by moving, flipping and/or sliding the pattern.

Graph #3

Translation Image



- Continue using the DISTFORM program to create the translation image. Several attempts may be necessary.
- Sketch your translation image on Graph #3 using a different color .
- Label the corresponding vertices S’, A’, and M’.
- Use TRACE to identify the coordinates of the new vertices to the nearest tenth.

Coordinates: S’_______   A’________   M’________

- Describe the movement of the pattern to get the translation image and compare the corresponding coordinates.

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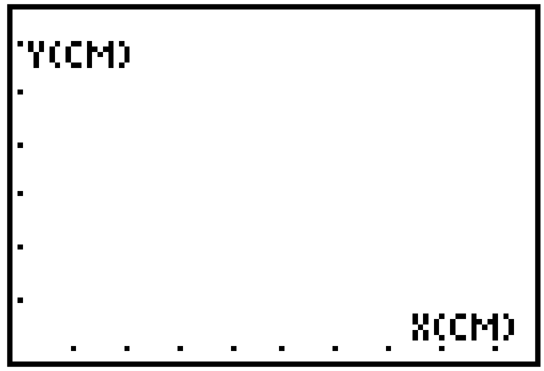
Activity #2

Create the rotation image.

- Discuss how to create a 30 degree rotation image about the origin by moving, flipping and/or sliding the pattern.

Graph #4

Rotation Image



- Continue using the DISTFORM program to create the rotation image. Several attempts may be necessary.
- Sketch your rotation image on Graph #4 using a different color .
- Label the corresponding vertices S', A', and M'.
- Use TRACE to identify the coordinates of the new vertices to the nearest tenth.

Coordinates: S'_______ A'________ M'________

- Describe the movement of the pattern to get the rotation image and compare the corresponding coordinates.

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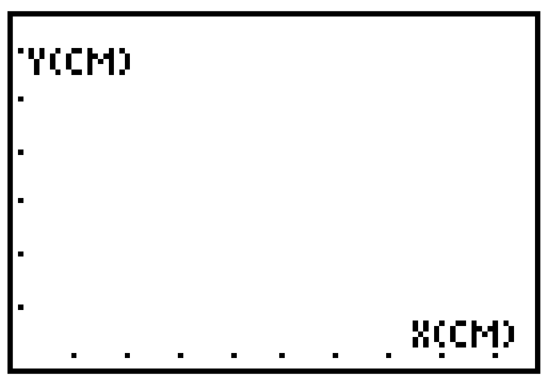
Create the glide reflection image.

- Discuss how to create a glide reflection image by moving, flipping and/or sliding the pattern.
- Choose a horizontal line to be used as the line of reflection. Write the equation of the line and draw the line on the graph below.

Graph #5

Glide Reflection Image

line of reflection: y = _____



Activity #2

- Continue using the DISTFORM program to create the glide reflection image. Several attempts may be necessary.
- Sketch your glide reflection image on Graph #5 using a different color .
- Label the corresponding vertices S', A', and M'.
- Use TRACE to identify the coordinates of the new vertices to the nearest tenth.

Coordinates: S'________   A'_________   M'_________

- Describe the movement of the pattern to get the glide reflection image and compare the corresponding coordinates.

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Prediction time.

1) If the pattern is an isosceles triangle whose base is on the x-axis, predict its reflection through a horizontal line. Sketch an isosceles triangle, choose a horizontal line and show the reflection image on graph #6 below.

Prediction:

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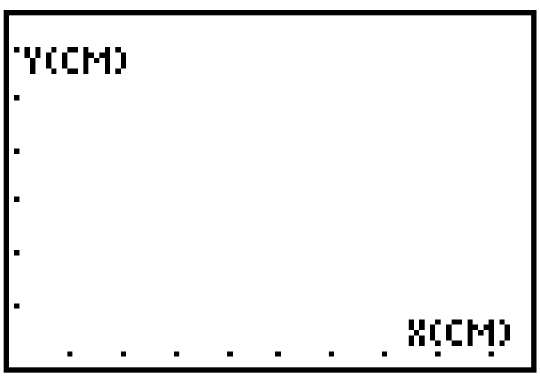
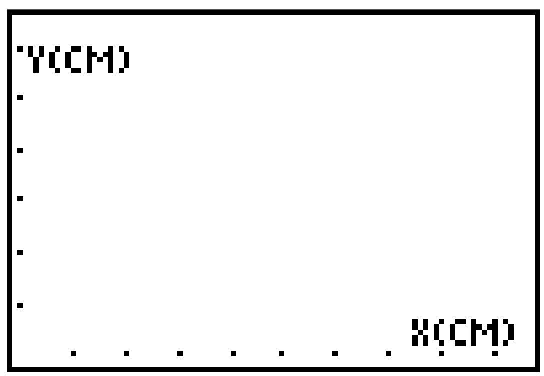
Graph #6



Graph #7



2) If the pattern is still an isosceles triangle whose base is on the x-axis, predict its 90 degree rotation image about its vertex (the top). Sketch the isosceles triangle, and show its rotation image on graph #7 above.

Prediction:

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Activity #2

- A dilation is a similarity transformation. Unlike the isometries where every image is congruent to its preimage, every image in a dilation is **similar** to its preimage. There are two types of dilations. If the image is larger than the original figure, the dilation is an **enlargement.** If the image is smaller than the original figure, the dilation is a **reduction.**

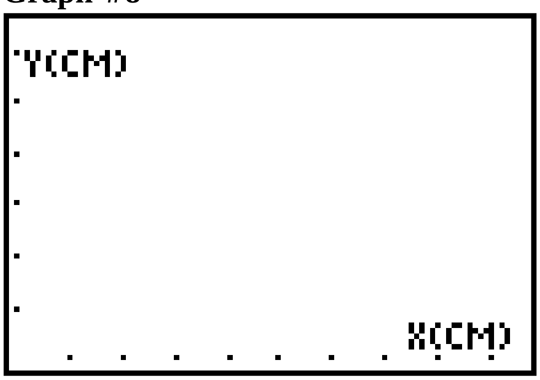
3) If the pattern is still an isosceles triangle whose base is on the x-axis, predict the relationships of the measurements of corresponding sides and angles. Sketch the isosceles triangle, show its enlargement on graph #8 and its reduction on graph #9 below.

Prediction:

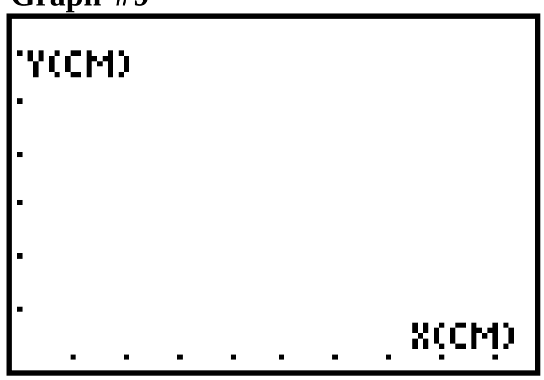
Sides____

Angles____

Graph #8



Graph #9



Activity #2

PATTERN SHEET